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Two Fluids Cosmological Models in Scale Covariant Theory of Gravitation

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ABSTRACT

The present paper deals with Bianchi type I two fluids cosmological model in scale covariant theory of gravitation.Matter fluid modeling observed matter and radiating fluid modeling cosmic microwave background radiation are taken as source. Exact Solutions of the field equations are obtained. Both interacting and non-interacting cases of two fluids are investigated. The exact solutions are obtained for constraints $X_1 = X_2 = X_3 = 0$. The energy densities are positive for the negative value of parametric constant α in case of exponential model. Energy transfer from matter to radiation is observed in case of interacting fluid. Some physical parameter of the obtained model is discussed in detail.

Keywords: Two fluids, Bianchi type I, Scale covariant theory.

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The participation of each author corresponds to the criteria of authorship and contributorship emphasized in the Recommendations for the Conduct, Reporting, Editing, and Publication of Scholarly work in Medical Journals of the International Committee of Medical Journal Editors. Indeed, all the authors have actively participated in the redaction, the revision of the manuscript, and provided approval for this final revised version.

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1. INTRODUCTION

Cosmology is the study of the universe as a whole. The general theory of relativity provides basic tools for constructing cosmological models of the universe. It is generally acclaimed as a mathematically precise and physically sound theory of gravitation. However, in recent years, there has been a lot of interest in several alternative theories of gravitation. Brans-Dicke (BD) theory [1] is one of the noteworthy among the various modification of general relativity. BD theory introduces a dynamical scalar field to account for variable gravitational constant G. Nordtvedt [2] proposed a general class of scalar-tensor theories in which the parameter w of the BD theory is allowed to be an arbitrary function of the scalar field. In SaezBallester's theory [3] metric is coupled with a dimensionless scalar field. Like BD theory, there is another viable alternative to general relativity which admits a variable G proposed by Canuto et al.[4]. The cosmological constant appears as avariable parameter in the framework of scale covariant theory. In scale covariant theory, Einstein's field equations are valid in gravitational units, whereas physical quantities are measured in the atomic units. The metric tensors in the two systems of units are related by a conformal transformation.

$$\bar{g}_{ij} = \varphi^2(x^k) g_{ij} \tag{1}$$

where a bar denotes gravitational units and unbarred denotes atomic units. An important feature of this theory is that no independent equation for ϕ exists.Beesham [5], Venkateswarlu[6], Reddy et al.[7], Ram et al.[8], Zeyanddin and Saha [9], Katore et al.[10] are some of the authors who have investigated several aspects of the scale covariant theory of gravitation.

Two fluid models, including energy densities of radiation and matter, are cosmologically important. Cosmological observations suggest that the radiation frame and the matter frame of the universe may not coincide [11]. The radiating fluid is modelinga cosmic microwave background. The matter-fluid modeling the observed matter content of the universe. Recently, researchers have beentaking keen interest in two fluids cosmological models. Amirhashchi et al.[12] have evaluated interacting two-fluid dark energy models inanon-flat universe. Khalatnikovet al. [13] have studied the quasi-isotropic expansion for a simple two-fluid cosmological models, including radiation and string gas. Coley and Dunn [14] have investigated the two fluids source of Bianchi type VI0 models. Pant and Oli [15] have examined the Bianchi type II space-time with a two-fluid cosmological model.

The paper is organized as follows: section 2 contains metric and field equations. Section 3 is devoted to solutions for non-interacting cases of fluids. In section 4, we present solutions in case of fluid interaction. In section 5, we conclude our obtained results.

2. METRIC AND FIELD EQUATIONS

Bianchi type space times I-IX play a vital role in understanding and describing of the early stages of the evolution of the universe. Bianchi type space times are homogeneous and anisotropic. The Friedmann-Robertson-Walker model is useful to describe the present day universe. However, the early universe may have been different than the present. Therefore, anisotropic space times are important. In this regard, we consider the Bianchi type I model in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 \tag{2}$$

Recently, two-fluid scenarios for the dark energy models in Brans-Dicke theory of gravitation have been evaluated by Reddy et al. [16]. Vishwakarma [17] investigated theBianchi type I models with varying gravitational constant(G) and cosmological constant (Λ). Bianchi type I two fluid models in the presence and absence of variable G and Λ is studied by Oli [18]. This motivates us to consider the Bianchi type I cosmological model in Scale covariant theory of gravitation. The field equations in the scale covariant theory with zero cosmological constant are

$$R_{ij} - \frac{1}{2}Rg_{ij} + f_{ij}(\varphi) = -8\pi G(\varphi)T_{ij} \tag{3}$$

$$\varphi^2 f_{ij}(\varphi) = 2\varphi\varphi_{;ij} - 4\varphi_{;i}\varphi_{;j} - \varphi_{ij}\left(2\varphi\varphi^k_{;k} - \varphi'^k\varphi_{,k}\right) \tag{4}$$

Here, φ is the scalar function (or gauge function) satisfying $0 < \varphi < \infty$ and other symbols have their usual meanings as in Riemannian geometry. The energy momentum tensor for two- fluid given by Letelier [19] and Bayin [20] is as follows

$$\{T_{ij} = (T^m)_{ij} + (T^r)_{ij} \tag{5}$$

here $(T^m)_{ij}$ is the energy momentum of matter field and $(T^r)_{ij}$ is the energy momentum tensor of the radiation field which is taken in the following form:

$$(T^m)_{ij} = (P_m + \rho_m)u_i^m u_j^m - P_m g_{ij} \tag{6}$$

$$(T^r)_{ij} = \frac{4}{3}\rho_r u_i^r u_j^r - \frac{1}{3}\rho_r g_{ij} \tag{7}$$

where ρ_m is the energy density of matter, P_m the pressure of the matter and ρ_r the energy density of radiation with $g^{ij}u_i^m u_j^m = 1, g^{ij}u_i^r u_j^r = 1$ and $u_i^m = (0,0,0,1), u_i^r = (0,0,0,1)$ then

$$T_1^1 = T_2^2 = T_3^3 = -\left(P_m + \frac{1}{3}\rho_r\right), T_4^4 = \rho_m + \rho_r \tag{8}$$

Using equations (2), (3), (4) and (8), the explicit field equations of the scale covariant theory can 73 be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{\varphi}}{A\varphi} - \frac{\dot{B}\dot{\varphi}}{B\varphi} - \frac{\dot{C}\dot{\varphi}}{C\varphi} - \frac{\ddot{\varphi}}{\varphi} + \left(\frac{\dot{\varphi}}{\varphi}\right)^2 = -8\pi G\left(P_m + \frac{1}{3}\rho_r\right) \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{A}\dot{\varphi}}{A\varphi} + \frac{\dot{B}\dot{\varphi}}{B\varphi} - \frac{\dot{C}\dot{\varphi}}{C\varphi} - \frac{\ddot{\varphi}}{\varphi} + \left(\frac{\dot{\varphi}}{\varphi}\right)^2 = -8\pi G\left(P_m + \frac{1}{3}\rho_r\right) \tag{10}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{\varphi}}{A\varphi} - \frac{\dot{B}\dot{\varphi}}{B\varphi} + \frac{\dot{C}\dot{\varphi}}{C\varphi} - \frac{\ddot{\varphi}}{\varphi} + \left(\frac{\dot{\varphi}}{\varphi}\right)^2 = -8\pi G\left(P_m + \frac{1}{3}\rho_r\right) \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{\varphi}}{A\varphi} - \frac{\dot{B}\dot{\varphi}}{B\varphi} - \frac{\dot{C}\dot{\varphi}}{C\varphi} + \frac{\ddot{\varphi}}{\varphi} - 3\left(\frac{\dot{\varphi}}{\varphi}\right)^2 = 8\pi G(\rho_m + \rho_r) \tag{12}$$

where overhead dot denotes differentiation with respect to t. We have four equations and $P_m, \rho_m, \rho_{r,}A, B, C, \phi, G$ eight unknowns. Field equations involving two fluids can be solved by taking interacting and non-interacting situations. Thenon-interacting phase is used to model the post-recombination era. In post-recombination era the photon got themselves free from the CMB. In theinteracting phases matter obeys the equation of state. The interactive phase describes the pre-recombination era where the photons were bound to the matter [21].

3. NON-INTERACTING MODELS

Using equations (9) and (10), we obtain

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\frac{\dot{C}}{C}+2\left(\frac{\dot{B}}{B}-\frac{\dot{A}}{A}\right)\frac{\dot{\varphi}}{\varphi}=0 \tag{13}$$

Equation (13) further reduces to

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)+\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-2\frac{\dot{\varphi}}{\varphi}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0 \tag{14}$$

Let

$$V=ABC \tag{15}$$

Making the use of equation (15) in equation (14), we yield

$$\frac{d}{dt}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)\left(2\frac{\dot{\varphi}}{\varphi}-\frac{\dot{V}}{V}\right) \tag{16}$$

Integrating equation (16), we get

$$\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=\frac{x_1\varphi^2}{V} \tag{17}$$

Equation (17) further reduces to

$$\frac{A}{B}=d_1\,exp\left(x_1\int\frac{\varphi^2}{V}dt\right) \tag{18}$$

Similarly using equations (9), (11) and equations (10), (11), we obtain

$$\frac{A}{C}=d_2\,exp\left(x_2\int\frac{\varphi^2}{V}dt\right) \tag{19}$$

$$\frac{B}{C}=d_3\,exp\left(x_3\int\frac{\varphi^2}{V}dt\right) \tag{20}$$

where $d_1, d_2, d_3, x_1, x_2, x_3$ are integration constant which satisfy the condition $d_2 = d_1 d_3, x_2 = x_1 + x_3$ of constant. From equations (18)-(20), we obtain

$$A=D_1V^{\frac{1}{3}}exp\left(X_1\int\frac{\varphi^2}{V}dt\right) \tag{21}$$

$$B=D_2V^{\frac{1}{3}}exp\left(X_2\int\frac{\varphi^2}{V}dt\right) \tag{22}$$

$$C=D_3V^{\frac{1}{3}}exp\left(X_3\int\frac{\varphi^2}{V}dt\right) \tag{23}$$

Where $D_1, D_2, D_3, X_1, X_2, X_3$ are integration constant which satisfies the condition of constant $D_1 D_2 D_3 = 1, X_1 + X_2 + X_3 = 0$. The Hubble parameter is given by $H = \frac{1}{3}(H_1 + H_2 + H_3)$ where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C}$ are directional Hubble parameters in the direction of $x,\ y,\ z$ axes, respectively.

$$H=\frac{1}{3}\frac{\dot{V}}{V}=\frac{\dot{R}}{R} \tag{24}$$

where R is the average scale factor.The special law of variation of Hubble parameter was proposed by Berman [22]. It gives the constant value of the deceleration parameter. It has the following form:

$$\boldsymbol{H = lR^{-n}} \tag{25}$$

where $> 0, n \geq 0$ are constant. The equations (24) and (25), give us

$$\boldsymbol{\dot{R} = lR^{-n+1}} \tag{26}$$

Recently, Singh and Kumar [23] have used the law of variation Hubble parameter to obtain solutions to field equations of general relativity. The deceleration parameter is obtained as

$$\boldsymbol{q = n - 1} \tag{27}$$

The sign of the deceleration parameter indicates acceleration or deceleration of the universe. Negative signs of q indicate accelerating, whereas the positive sign of q indicates a decelerating universe [24]. From equation (27), it is clear that for n<1 the q is negative i.e., the model is accelerating. For n>1, the sign of q is positive; the model is decelerating. For n = 1, the universe expands with acceleration.
To obtain the exact solution of the field equations, we consider the scale function as

$$\boldsymbol{\varphi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon}, \varepsilon = \pm 1, \pm\frac{1}{2}} \tag{28}$$

where t_0 is constant. The most suitable form of gauge function to fit the observational data is given by

$$\boldsymbol{\varphi(t) \sim t^{\frac{1}{2}}} \tag{29}$$

The gravitational term G is assumed as time-dependent. In the literature, we have found that G is a decreasing function of time. However, the possibility of increasing the function of time is also not ruled out. We assume the most useful and simple form of G as

$$\boldsymbol{G = \alpha t} \tag{30}$$

where α is the proportionality constant. It gives us a physically viable and realistic model of the universe. Levit [25] and Beesham [26] have considered the proportionality relation $G \propto t^{\lambda}$, whereas Sistero [27] has taken the relation $G \propto R^{\lambda}$ to find pressure and energy density in terms of time.

Case I) when n =0

In this case, taking $n = 0$ in equation (26), we get the average scale factor as

$$\boldsymbol{R = c_1 \, exp(\, lt)} \tag{31}$$

The volume of the universe is found to be

$$\boldsymbol{V = exp(\, 3lt)} \tag{32}$$

From equation (32), it is clear that the volume of the universe is an exponential function of time. At $t = 0$ it is non-zero i.e., the universe starts to expand form from constant volume and tends to infinity with increasing infinite time. From equations (21)-(23), we have the following solutions

$$\boldsymbol{A = D_1 \, exp\left(lt - \frac{X_1}{l^2}(lt+1)\, exp(-lt)\right)} \tag{33}$$

$$\boldsymbol{B = D_2 \, exp\left(lt - \frac{X_2}{l^2}(lt+1)\, exp(-lt)\right)} \tag{34}$$

$$\boldsymbol{C = D_3 \, exp\left(lt - \frac{X_3}{l^2}(lt+1)\, exp(-lt)\right)} \tag{35}$$

These are the solutions of the field equations subjected to the condition $X_1 = X_2 = X_3 = 0$. For this condition, the values of the metric potentials A, B, C are differ by constants. It is free from the initial singularity. The energy density of matter, energy density of radiation, matter density parameter radiation density parameter, and energy density parameter have the following expressions:
The density of the matter

$$\boldsymbol{\rho_m = \frac{3l^2}{4\pi\alpha(4-3\gamma)t} + \frac{1}{16\pi\alpha(4-3\gamma)t^3} - \frac{3l}{8\pi\alpha(4-3\gamma)t^2}} \tag{36}$$

The density of the radiation

$$\rho_r = \frac{9(\gamma-2)l^2}{8\pi\alpha(4-3\gamma)t} - \frac{3(3-2\gamma)}{16\pi\alpha(4-3\gamma)t^3} - \frac{3l(3\gamma-2)}{16\pi\alpha(4-3\gamma)t^2} \tag{37}$$

Matter density parameter

$$\Omega_m = \frac{\rho_m}{3H^2} = \frac{1}{4\pi\alpha(4-3\gamma)t} + \frac{1}{48l^2\pi\alpha(4-3\gamma)t^3} - \frac{1}{8\pi\alpha l(4-3\gamma)t^2} \tag{38}$$

Radiation density parameter

$$\Omega_r = \frac{\rho_r}{3H^2} = \frac{3(\gamma-2)}{8\pi\alpha(4-3\gamma)t} - \frac{(3-2\gamma)}{16\pi l^2\alpha(4-3\gamma)t^3} - \frac{(3\gamma-2)}{16\pi l\alpha(4-3\gamma)t^2} \tag{39}$$

Energy density parameter

$$\Omega = \Omega_m + \Omega_r = \frac{(3\gamma-4)}{8\pi\alpha(4-3\gamma)t} + \frac{(6\gamma-8)}{48l^2\pi\alpha(4-3\gamma)t^3} + \frac{(3\gamma-4)}{16\pi\alpha l(4-3\gamma)t^2} \tag{40}$$

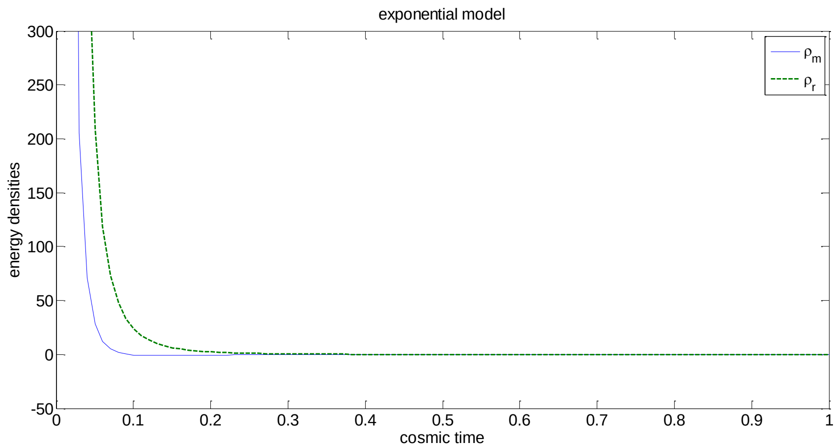

Figure 1: Plot of matter and radiation energy densities versus cosmic time.

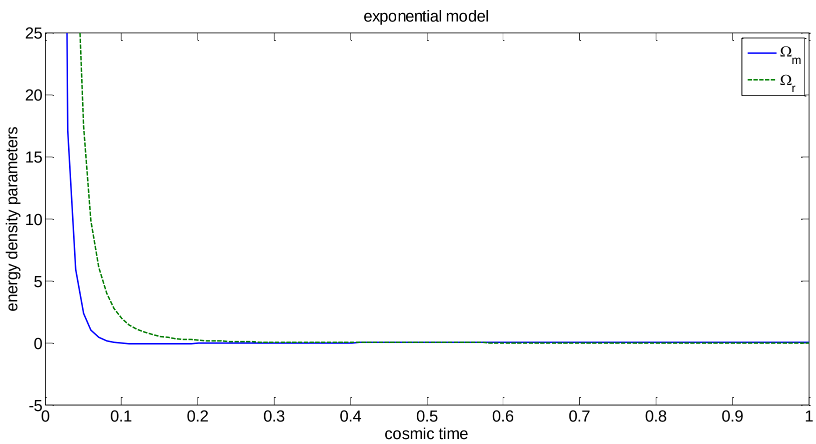

Figure 2: Plot of matter and radiation energy density parameters versus cosmic time.

The energy density of matter and radiation are decreasing functions of time t. Neat $t = 0$, ρ_m and ρ_r are large. As $t \to \infty$ both $\rho_m, \rho_r \to 0$. Near $t = 0$, $\rho_m < \rho_r$. One important point is that the energy densities are positive for the negative value of α. The fluid is Zeldovich i.e.,$\gamma = 2$. This indicates the role of gravitational constant G. It is zero at $t = 0$ and gradually decreases to minus infinity as $t \to \infty$. From figure (1), it is also clear that matter density is smaller than radiation density and $\rho_m \to 0$ earlier than $\rho_r \to 0$. From figure (2), we observe that the matter and radiation energy density parameters behave like energy densities. As the universe is radiation-dominated in the early epoch and matter-dominated at in the present. The result is as per expectation.

Case II) when $n \neq 0$

In this case, equation (26) leads to the following value of average scale factor

$$R = (nlt)^{\frac{1}{n}} \tag{41}$$

The volume of the universe is found to be

$$V = (nlt)^{\frac{3}{n}} \tag{42}$$

From equation (21)-(23), we obtain following values of metric potentials A, B, C:

$$A = D_1 (nlt)^{\frac{1}{n}} exp\left(\frac{nX_1}{(2n-3)(nl)^{\frac{3}{n}}} t^{2-\frac{3}{n}}\right) \tag{43}$$

$$B = D_2 (nlt)^{\frac{1}{n}} exp\left(\frac{nX_2}{(2n-3)(nl)^{\frac{3}{n}}} t^{2-\frac{3}{n}}\right) \tag{44}$$

$$C = D_3 (nlt)^{\frac{1}{n}} exp\left(\frac{nX_3}{(2n-3)(nl)^{\frac{3}{n}}} t^{2-\frac{3}{n}}\right) \tag{45}$$

These are the solutions of the field equations subjected to the condition $X_1 = X_2 = X_3 = 0$. As like in case I, the metric potentials A, B, C differ by constant and we get the same constraint. The energy density of matter, energy density of radiation, matter density parameter, radiation density parameter, and energy density parameter have following expressions:

The density of the matter

$$\rho_m = \frac{24 - 18n + n^2}{16\pi\alpha n^2 (4 - 3\gamma) t^3} \tag{46}$$

The density of the radiation

$$\rho_r = \frac{9\gamma n - 18\gamma + 6\gamma n^2 + 6n - 11n^2}{16\pi\alpha n^2 (4 - 3\gamma) t^3} \tag{47}$$

The matter density parameter

$$\Omega_m = \frac{24 - 18n + n^2}{48\pi\alpha(4 - 3\gamma)t} \tag{48}$$

The radiation density parameter

$$\Omega_r = \frac{9\gamma n - 18\gamma + 6\gamma n^2 + 6n - 11n^2}{48\pi\alpha(4 - 3\gamma)t} \tag{49}$$

The energy density parameter

$$\Omega = \frac{9\gamma n - 18\gamma + 6\gamma n^2 + 24 - 12n - 10n^2}{48\pi\alpha(4 - 3\gamma)t} \tag{50}$$

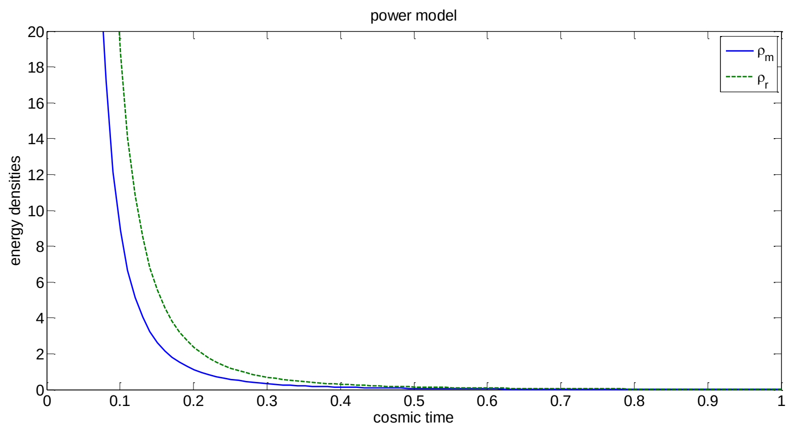

Figure 3: Plot of matter and radiation energy densities versus cosmic time.

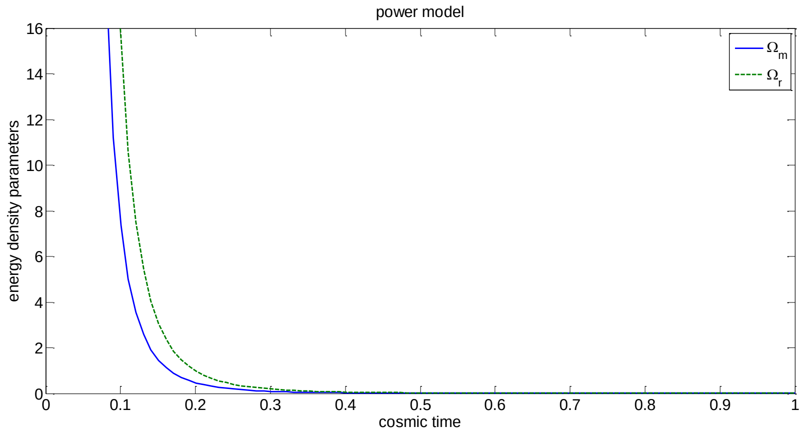

Figure 4: Plot of matter and radiation energy density parameters versus cosmic time.

From equation (46), it is clear that ρ_m is positive for $\frac{24+n^2}{n} > 18$ and $\frac{4}{3} > \gamma \geq 1$. Also, ρ_m is positive for $\frac{24+n^2}{n} < 18$ and $\frac{4}{3} < \gamma \leq 2$. When $\frac{24+n^2}{n} > 18$ and $\frac{4}{3} < \gamma \leq 2$ we need to take negative value of α for viable model. Further when $\frac{24+n^2}{n} < 18$ and $\frac{4}{3} > \gamma \geq 1$, we also need to take α negative for the viable model. For $\frac{24+n^2}{n} = 18, \rho_m = 0$. The energy density of matter and radiation are decreasing functions of time. Near $t = 0$, both ρ_m and ρ_r are large. As $t \to \infty$, both ρ_m and ρ_r tend to zero. From figure (3), it is seen that matter density is smaller than radiation density. ρ_m and $\rho_r$tend to zero for large values of t. Further, the matter and radiation density parameters are behaving like densities (figure 4). Initially, the radiation-dominated universe tends to matter-dominated, which is as per expectation.

4. INTERACTING MODEL

Here we consider the interaction between matter and radiation densities. Therefore, they conservation equation behave as follows:

$$\dot{\rho}_m + 3\frac{\dot{R}}{R}(P_m + \rho_m) = Q \tag{51}$$

$$\dot{\rho}_r + 4\frac{\dot{R}}{R}\rho_r = -Q \tag{52}$$

We follow Amendola [28] and Guo [29] for taking relations of interacting terms in the form $Q = 3H\sigma\rho_m, \sigma$ is a coupling constant. If the interacting term Q is positive, then there exists energy transfer from radiation fluid to matter fluid. If the interacting term Q is negative, then there is energy transfer from matter to radiation. If $Q = 0$ then, there is no interaction between in the fluids. We have a relation to Hubble parameter which gives us

$$H = \frac{\dot{R}}{R} \Rightarrow R = exp(Ht) \tag{53}$$

Using equations (51) and (53), we get

$$\rho_m = \rho_0\, exp(-3H(\gamma - \sigma)t) \tag{54}$$

In order to get a viable model, we have constraints $\gamma - \sigma < 0$ i.e., $\gamma < \sigma$. For these constraints, the interacting term Q is negative (figure 7). This indicates that energy transforms from matter to radiation. This observation is also noted for Bianchi type I in general relativity [30]. Adding equations (9), (10), (11) and three times equations (12), we obtain

$$\frac{\ddot{V}}{V} - 2\frac{\dot{V}\dot{\varphi}}{V\varphi} = -4\pi G(3P_m - 3\rho_m - 2\rho_r) \tag{55}$$

The radiation energy density is obtained as
The radiation density

$$\rho_r = -\frac{3H}{8\pi\alpha t^2} + \frac{9H^2}{8\pi\alpha t} - \frac{3(\gamma - 2)\rho_0}{2\, exp(3H(\gamma - \sigma)t)} \tag{56}$$

To find the value of Hubble parameter we assume

$$\rho\alpha\theta^2 \Rightarrow \rho = 9mH^2 \tag{57}$$

Here m is constant. Then the value of Hubble parameter is found to be

$$H = -\frac{\frac{16\pi\alpha m}{3} exp\left(\frac{1}{16\pi m\alpha t}\right)}{\int \frac{1}{t} exp\left(\frac{1}{16\pi\alpha m t}\right) dt} \tag{58}$$

The matter, radiation and total energy density parameter have the following expressions:
The matter density parameter

$$\Omega_m = \frac{1}{3H^2}\rho_0\, exp(-3H(\gamma-\sigma)t) \tag{59}$$

The radiation density parameter

$$\Omega_r = -\frac{1}{8\pi H\alpha t^2} + \frac{3}{8\pi\alpha t} - \frac{(\gamma-2)\rho_0}{2H^2\, exp(3H(\gamma-\sigma)t)} \tag{60}$$

The energy density parameter

$$\Omega = -\frac{1}{8\pi H\alpha t^2} + \frac{3}{8\pi\alpha t} + \frac{(8-3\gamma)\rho_0}{6H^2\, exp(3H(\gamma-\sigma)t)} \tag{61}$$

From figures (5) and (6), it is clear that the energy density of matter is smaller than the energy density of radiation. The energy density of matter is constant throughout evolution, whereas the energy density of radiation is a decreasing function of time. The energy density of radiation is large initially and tends to be constant at infinite time. Matter density parameter starts to increase from fix-constant value to its maximum and remains constant with increasing time. Radiation density parameter is a decreasing function of time. In the case of Bianchi type II viable models are possible only for $\gamma = 1$ [21].

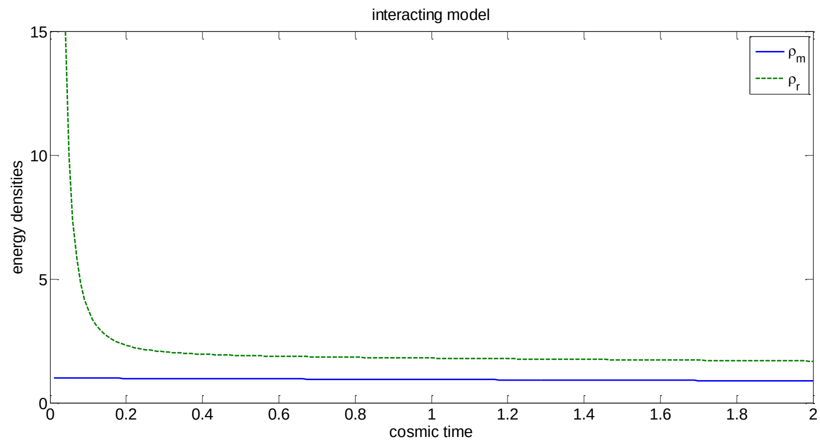

Figure 5: Plot of matter and radiation energy densities versus cosmic time.

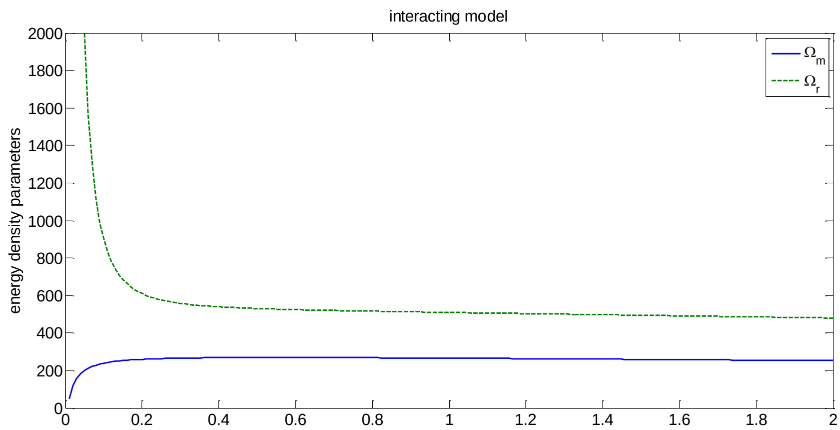

Figure 6: Plot of matter and radiation energy density parameters versus cosmic time.

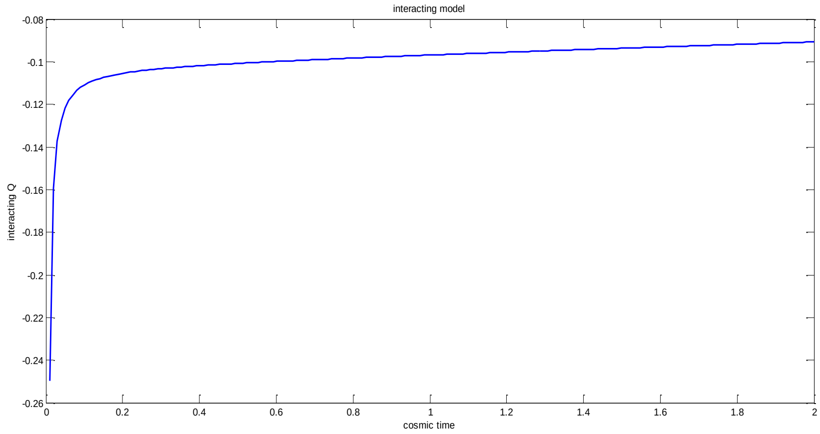

Figure 7: Plot of interacting term Q versus cosmic time.

CONCLUSION

In this investigation, we have explored matter and radiation fluids in the scale covariant theory of gravitation for Bianchi type I. Both interacting and non-interacting possibilities are examined. In the non-interacting models, two cases are analyzed according to two different forms of Hubble parameters. In both cases I and II, we get a constraint $X_1 = X_2 = X_3 = 0$ on solutions. These types of constrints are not found in case of the general theory of relativity [18]. Matter and radiation energy densities are decreasing functions of time. A similar nature of decreasing functions of ρ_m and ρ_r is also obtained by Pant and Oli [15], Adhav, et al.[21]. In case I, the energy densities are positive only for the negative value of constant α. The stability of the model is analyzed using the square of the sound speed function $C_s^2 = \frac{dP}{d\rho}$.

The model is stable when the function is positive. Figure (8), shows that the behavior of the speed of sound function C_s^2 is positive. Hence, stability occurs throughout the evolution of the universe.

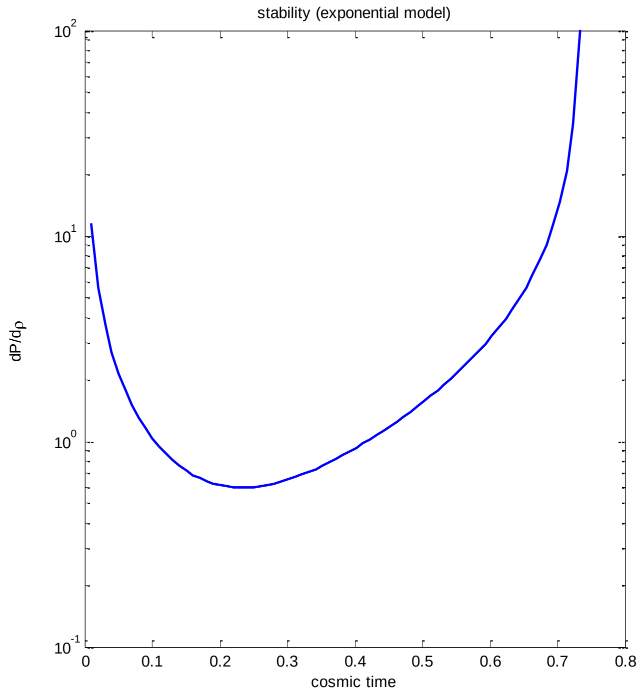

Figure 8. Plot of $\frac{dP}{d\rho}$ with respect ot cosmic time

In case II, the energy density of matter is positive for $\frac{24+n^2}{n} > 18$ and $\frac{4}{3} > \gamma \geq 1$ as well as for $\frac{24+n^2}{n} < 18$ and $\frac{4}{3} < \gamma \leq 2$. Coley and Dunn [14] in the case of Bianchi type VI_0 have found that $\rho_m = 0$ in power-law model. Oli [18] has found that the energy density of radiation is negative at latter time in the power-law model. Which is not found in the scale covariant theory.

In the case of interaction of two fluids we have constraints $\gamma - \sigma < 0$ i.e., $\gamma < \sigma$. There is energy transformation from matter to radiation. A similar type of observation is also noted in general relativity [30].

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